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Non-Abelian Collective Excitations in Unlinearized Quark-Gluon Plasma Media

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Abstract

We study the effect of unlinearized medium on the collective excitations in quark-gluon plasma. We present two kinds of non-Abelian oscillation solutions which respectively correspond to weakly and strongly nonlinear coupling of field components in color space. We also show that the weakly nonlinear solution is similar to Abelian-like one but has the frequency shift, which is of order $g^2 T$, from eigenfrequency.

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Collective excitations play an important role in the analyses of static and dynamic properties of a many-particle system such as a plasma. It is an extensively well-studied subject in electromagnetic plasma[1, 2]. The quark-gluon plasma oscillation modes have been studied for a long time in the framework of finite-temperature(FT)QCD[3] and in kinetic-theory approach[4]. First of all, most studies are limited to the linearized equations for both Yang-Mills field equations and the kinetic equations[4], where the excitations reduce to Abelian-like plasma waves. Up to 1994, Blaizot and Iancu gave the non-Abelian oscillation solutions at leading order in the gauge coupling g approximation, where the medium still remains to be Abelian-like[5] although the mean field equations are unlinearized. The purpose of this Letter is to present new non-Abelian solutions that we obtained recently in true non-Abelian the medium.

It is well known that the equations satisfied by the gauge mean fields $A_a^\mu(x)$ in a medium [Throughout this work, the greek indices refer to Minkovski space, while the Latin subscripts

indicate color indices for the generators of the gauge group $SU(N)$ are

$$D^\mu F_{\mu\nu}^a = j_\nu^{\text{inda}}(x), \quad (1)$$

where $D^\mu = \partial^\mu + igA^\mu(x)$, $A^\mu \equiv A_a^\mu \tau^a$, and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - igf_{abc}\tau^c A_a^\mu A_b^\nu \tau_a$, the generator of $SU(N)$. The energy density of the collective excitation in the plasma can be written as

$$T^{00}(x) \equiv T_{\text{YM}}^{00}(x) + t^{00}(x), \quad (2)$$

with $\partial_t t^{00}(x) = -\mathbf{E}_a \cdot \mathbf{j}^{\text{inda}}(x)$, where $T^{00}(x)$ is the standard Yang-Mills contribution[6]. The induced current j_ν^{ind} describes the response of the plasma to the color gauge fields A_a^μ , which relates to the fluctuations in the phase-space color densities of quarks and gluons. It is expressed as

$$\begin{aligned} j_\nu^{\text{inda}} &= g \int \frac{d^3 p}{(2\pi)^3} v_\mu \{ N_f [Q_+^{(n)a}(\mathbf{p}, x) - Q_-^{(n)a}(\mathbf{p}, x)] \\ &+ \text{tr}[T_a G^{(n)}(\mathbf{p}, x)] \}, \end{aligned} \quad (3)$$

where the quark(antiquark) and gluon distribution functions ($Q_\pm(\mathbf{p}, x)$ and $G(\mathbf{p}, x)$) are governed by the following kinetic equations, which describe non-Abelian plasma medium[7].

$$p^\mu D_\mu Q_\pm(\mathbf{p}, x) \pm \frac{g}{2} p^\mu \partial_p^\nu \{ F_{\mu\nu}(x), Q_\pm(\mathbf{p}, x) \} = 0, \quad (4)$$

$$p^\mu \mathcal{D}_\mu G(\mathbf{p}, x) + \frac{g}{2} p^\mu \partial_p^\nu \{ \mathcal{F}_{\mu\nu}(x), G(\mathbf{p}, x) \} = 0, \quad (5)$$

where the calligraphic letters represent the corresponding operators in adjoint representation of $SU(N)$, the generator denoted by T_a . From these equations we can obtain the fluctuations around the average phase-space color densities of plasma constituents. For convenience, we choose the temporal gauge later on, $A_a^0 = 0$. Therefore, for the solutions which are uniform in space we have

$$\partial_t Q_\pm^1 = \mp g \mathbf{v} \cdot \partial_t \mathbf{A} \frac{dQ_\pm^0}{dp}, \quad (6)$$

$$\partial_t G^1 = -g \mathbf{v} \cdot \partial_t \mathbf{A} \frac{dG^0}{dp}, \quad (7)$$

$$\begin{aligned} \partial_t Q_\pm^2 &= \mp ig^2 [\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}] \frac{dQ_\pm^0}{dp} \\ &\pm \frac{g^2}{2} \{ \mathbf{v} \cdot \partial_t \mathbf{A}, \mathbf{v} \cdot \mathbf{A} \} \frac{d^2 Q_\pm^0}{dp^2}, \end{aligned} \quad (8)$$

$$\begin{aligned} \partial_t G^2 &= \mp ig^2 [\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}] \frac{dG^0}{dp} \\ &+ \frac{g^2}{2} \{ \mathbf{v} \cdot \partial_t \mathbf{A}, \mathbf{v} \cdot \mathbf{A} \} \frac{d^2 G^0}{dp^2}, \end{aligned} \quad (9)$$

$$\begin{aligned}
\partial_t Q_{\pm}^3 &= \pm g^3 [\mathbf{v} \cdot \mathbf{A}, \int dt [\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}]] \frac{dQ_{\pm}^0}{dp} \\
&+ \frac{ig^3}{4} [\mathbf{v} \cdot \mathbf{A}, \{\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}\}] \frac{d^2 Q_{\pm}^0}{dp^2} \\
&+ \frac{ig^3}{2} \{\mathbf{v} \cdot \partial_t \mathbf{A}, \int dt [\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}]\} \frac{d^2 Q_{\pm}^0}{dp^2} \\
&\mp \frac{g^3}{8} \{\mathbf{v} \cdot \partial_t \mathbf{A}, \{\mathbf{v} \cdot \mathbf{A}, \mathbf{v} \cdot \mathbf{A}\}\} \frac{d^3 Q_{\pm}^0}{dp^3}, \tag{10}
\end{aligned}$$

$$\begin{aligned}
\partial_t G^3 &= +g^3 [\mathbf{v} \cdot \mathcal{A}, \int dt [\mathbf{v} \cdot \mathcal{A}, \mathbf{v} \cdot \mathcal{A}]] \frac{dG^0}{dp} \\
&+ \frac{ig^3}{4} [\mathbf{v} \cdot \mathcal{A}, \{\mathbf{v} \cdot \mathcal{A}, \mathbf{v} \cdot \mathcal{A}\}] \frac{d^2 G^0}{dp^2} \\
&+ \frac{ig^3}{2} \{\mathbf{v} \cdot \partial_t \mathcal{A}, \int dt [\mathbf{v} \cdot \mathcal{A}, \mathbf{v} \cdot \mathcal{A}]\} \frac{d^2 G^0}{dp^2} \\
&- \frac{g^3}{8} \{\mathbf{v} \cdot \partial_t \mathcal{A}, \{\mathbf{v} \cdot \mathcal{A}, \mathbf{v} \cdot \mathcal{A}\}\} \frac{d^3 G^0}{dp^3}, \tag{11}
\end{aligned}$$

All previous studies in this field were limited to linear induced current in the linear response approximation[5, 6, 8, 9]. From Eq(3),(6) and (7), we easily write the current as[5, 6]

$$\mathbf{j}_a^l = -\omega_p^2 \mathbf{A}^a(t), \tag{12}$$

where $\omega_p^2 \equiv \frac{1}{3} \frac{g^2}{2\pi^2} \int dp p^2 \frac{\partial}{\partial p} [N_f(Q_+^0 + Q_-^0) + 2NG^0]$ is the plasma frequency when Q_{\pm}^0 and G^0 take the Fermi-Dirac and Bose-Einstein distributions respectively. As we have argued above, the result in the linear approximation is Abelian-like. We need to consider non-linear response of the plasma in order to get non-Abelian effect of the medium. Therefore, we also derive the non-linear induced currents from the non-linear density fluctuations (8),(9),(10) and (11). However, we are able to neglect the second-order current in contrast to the third-order current[10] (In fact, we can exactly prove that the even-order currents are always zero). Thus, the third-order current is the non-linear leading order contribution(We only consider the lowest order here), which is calculated according to the formulae below

$$\begin{aligned}
\mathbf{j}_a^n &= \text{tr}(2\tau^a \mathbf{J}_Q) + \text{tr}(T^a \mathbf{J}_G), \\
\mathbf{J}_Q &= \mathbf{J}_{Q_+} - \mathbf{J}_{Q_-} \\
&= \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} N_f (Q_+^3 - Q_-^3), \\
\mathbf{J}_G &= \int \frac{d^3 p}{(2\pi)^3} \mathbf{v} G^3. \tag{13}
\end{aligned}$$

Both linear and non-linear density fluctuations contributing to field equations(1) and

energy density(2), we have

$$\frac{d^2 A_i}{dt^2} + \omega_p^2 A_i + g^2 [[A_i, A_j], A_j] + j_i^n = 0, \quad (14)$$

and

$$\begin{aligned} T^{00}(t) &= \frac{1}{2} \left(\frac{d\mathbf{A}_a}{dt} \cdot \frac{d\mathbf{A}_a}{dt} + \omega_p^2 \mathbf{A}_a \cdot \mathbf{A}_a + \int dt \frac{d\mathbf{A}_a}{dt} \cdot \mathbf{j}_a^n \right) \\ &+ \frac{g^2}{4} f^{abc} f^{ade} (\mathbf{A}_b \cdot \mathbf{A}_d) (\mathbf{A}_c \cdot \mathbf{A}_e). \end{aligned} \quad (15)$$

Note that T^{00} is a conserved quantity and manifestly positive. We restrict ourselves to such configurations for the case of SU(2), and assume, for simplicity, that $A_a^i(t) = \mathcal{C}_a^i h^i(t)$ (no summation over i), taking constant $\mathcal{C} = \delta_a^i$ as did in previous works[5, 6]. This ansatz is what proposed by Baseyan, Matinyan and Savvidy[11]. We calculate and immediately find only the last terms of the right-hand side of equations (10) and (11) contribute to the current \mathbf{j}_a^n under the ansatz. We obtain

$$\mathbf{j}_a^n = -\frac{g^2}{2} \omega_{p'}^2 (3h_i^3 + h_i \sum_{j \neq i} h_j^2), \quad (16)$$

where $\omega_{p'}^2 = \frac{g^2}{180} \int \frac{p^2 dp}{2\pi^2} \frac{\partial^3}{\partial p^3} [N_f (Q_+^0 + Q_-^0) + 8NG^0]$ represents non-linear collective effect of plasma. The field equations(14) then become

$$\begin{aligned} \frac{d^2 h_i}{dt^2} &+ \omega_p^2 h_i + g^2 h_i \sum_{j \neq i} h_j^2 \\ &+ \frac{g^2}{2} \omega_{p'}^2 (3h_i^3 + h_i \sum_{j \neq i} h_j^2) = 0, \end{aligned} \quad (17)$$

These equations contain not only the linear but also non-linear thermal mass terms. They are similar to the past equations coupled nonlinearly with each other. However, it is more important that they differ, as we shall see, by the presence of self-coupled thermal term $\frac{g^2}{2} \omega_{p'}^2 3h_i^3$ from the effect of the non-linear response of plasma. The energy density associated with h_i is

$$\begin{aligned} T^{00} &= \frac{1}{2} \sum_i \left[\left(\frac{dh_i}{dt} \right)^2 + \frac{g^2}{2} h_i^2 \sum_{j \neq i} h_j^2 \right. \\ &\quad \left. + \omega_p^2 h_i^2 + \frac{g^2}{2} \omega_{p'}^2 \left(\frac{3}{2} h_i^4 + h_i^2 \sum_{j \neq i} h_j^2 \right) \right], \end{aligned} \quad (18)$$

where the effective Hamiltonian is composed of two terms of field contributions and two terms of thermal effects.

We see that the simplest motion is one dimensional. Let us consider now the solutions to equations (17) for the motion. Then the equations(17) reduce to for $h_1 = h, h_2 = h_3 = 0$

$$\frac{d^2h}{dt^2} + \omega_p^2 h + \frac{g^2}{2} \omega_p^2 3h^3 = 0, \quad (19)$$

We discuss now it for two cases: nearly equilibrium and far from equilibrium.

(i) First of all, we rewrite the non-linear self-coupled term in(19) as $\frac{g^2}{2} \omega_p^2 3\langle h^2 \rangle h$, where we replace h^2 by its time average $\langle h^2 \rangle$ which is so called fluctuation correlation intensity. We know correlation is small at the nearby equilibrium state of system. Therefore, the non-linear term is smaller than the linear term in(19) because the self-coupled term is two higher orders in the coupling constant g than the linear term. We replace then equation(19) by the following Iterating equations

$$\frac{d^2h^{(0)}}{dt^2} + \omega_p^2 h^{(0)} = 0, \quad (20)$$

$$\frac{d^2h}{dt^2} + [\omega_p^2 + \frac{g^2}{2} \omega_p^2 3(h^{(0)})^2]h = 0, \quad (21)$$

We have oscillation solution to the equations for $\phi = 0$ at initial time as follows

$$h^{(0)}(t) = \sqrt{2}\theta \cos \omega_p t, \quad (22)$$

$$h(t) = h_\theta^w \cos [\omega_\theta t(1 + \frac{3}{4} \omega_p^2 \theta^2) + \frac{3}{8} \omega_p^2 \theta^2 \sin 2\omega_p t], \quad (23)$$

where $\theta^2 = \frac{g^2 T^{00}}{\omega_p^4}$ is the dimensionless Hamiltonian, $h_\theta^w = \sqrt{2}\theta(1 + \frac{3}{2} \omega_p^2 \cos^4)^{\frac{1}{2}} \omega_p t$. $h(t)$ is weakly nonlinear and there exists an frequency shift from the eigenvalue because of nonlinear effect of plasma

$$\begin{aligned} \Delta\omega &= \omega_\theta - \omega_p \\ &= \frac{3}{4} \omega_p^2 \theta^2 (1 + \frac{1}{2\omega_p t} \sin 2\omega_p t) \omega_p, \end{aligned} \quad (24)$$

We have $\Delta\omega = \frac{3}{4} \omega_p^2 \theta^2 \omega_p$ to be of order $g^2 T$ coincided with previous works[12] when $\langle (h^{(0)})^2 \rangle$ replaces $(h^{(0)})^2$.

(ii)For far from equilibrium state, the self-coupled nonlinear term in (19) shall not be small when $\langle h^2 \rangle$ is large enough. In fact, it is possible to let $\langle h^2 \rangle \gg 1, \omega_p^2 \langle h^2 \rangle \sim 1$, such as turbulent plasma[13]. Of course, one expects there exist turbulent QGP, which has been roughly discussed in other works[10], in heavy ion collisions[9]. Equation(19) is strong nonlinear for the case and we know a Jacobi elliptic cosine solution to it,

$$h(t) = h_\theta^s \operatorname{cn}[\Gamma^{-\frac{1}{2}} (2\theta^2 + \Gamma^2)^{\frac{1}{4}} (\omega_p t - \phi); k], \quad (25)$$

with the $h_\theta^s = \frac{\omega_p}{g} [\Gamma(\sqrt{2\theta^2 + \Gamma^2} - \Gamma)]^{\frac{1}{2}}$, the modulus $k = \frac{1}{\sqrt{2}}(1 - \frac{\Gamma}{\sqrt{2\theta^2 + \Gamma^2}})^{\frac{1}{2}}$ and a period $T = \frac{4}{\omega_p} \frac{\Gamma^{\frac{1}{2}}}{(2\theta^2 + \Gamma^2)^{\frac{1}{4}}} K(k)$, where $\Gamma^2 = \frac{2}{3\omega_p^2}$ and $K(k)$ is the complete elliptic integral modulus k . What we have obtained here is an one dimensional solution similar to the two dimensional case in ref[5]. The factor Γ characterizes the nonlinearity and the non-Abelian property of the solution. The solution reduces to the Abelian-like harmonic oscillation[5, 6] and $T \rightarrow T_0 = 2\pi/\omega_p$ when $\theta \ll 1$, $k \rightarrow 0$ and $\text{cn} \rightarrow \cos$.

In summary, we have studied here the non-Abelian oscillations considering the nonlinear response of the plasma. We have shown that self-coupled thermal term play a vital role for non-Abelian excitations. We have obtained nonlinear solutions for SU(2) in one dimensional color space. We have also found a weak nonlinear solution similar to the Abelian-like harmonic oscillation but having a frequency shift from eigenfrequency ω_p for small correlation intensity and a strong nonlinear solution, which is the typical non-Abelian excitation[5, 6], for the inverse case. As Blaizot and Iancu[5] have pointed out, such as solutions for SU(2) can be embedded in any larger SU(N) theory in the standard way[14].

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